IASSC

Examination Reference Document

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Formulas

Statistical Tables

Descriptive Statistics

Calculation	Formula	Notes
Population Mean	$\mu = \frac{\sum X}{N}$	Where μ = Population Average X = Individual Values of Population N = Count of Individual Values
Sample Mean	$\overline{X} = \frac{\sum X}{n}$	Where $\overline{X}$ = Sample Average X = Individual Values of Population n = Count of Individual Values in Sample
Weighted Mean	$\overline{X}_w = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_nx_n}{w_1 + w_2 + w_3 + \ldots + w_n}$	Where $\overline{X}_W$ = Weighted Sample Average W_J = Weight of Value J XJ = Individual value to be weighted
Sample Mean of grouped data	$\overline{X} = \frac{\sum f_i X_i}{n}$	Where f_i = Number of observations In the i'th group X_i = Midpoint of the i'th class n = Count of all observations of i'th classes
Range	*Range = highest - lowest*	Where highest = Sample Max Value lowest = Sample Min Value
Mean Deviation	$MD = \frac{\sum \lvert X - \overline{X} \rvert}{n}$	Where X = Individual value in the sample $\overline{X}$ = Sample average n = Count of samples
Population Variance	$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$	Where μ = Population average X = Individual value in population N = Count of population

Calculation	Formula	Notes
Population Standard Deviation	$\sigma = \sqrt{\frac{\sum (X - \mu)^2}{N}}$	Where μ = Population average X = Individual value in population N = Count of population
Sample Variance	$s^2 = \frac{\sum (X - \overline{X})^2}{n-1}$	Where $\overline{X}$ = Sample average X = Individual value from sample n = Count of sample
Sample Standard Deviation	$s = \left[\sqrt{\frac{n}{n-1}}\right]\sigma$	*none*

Hypothesis Testing

Calculation	Formula	Notes
2 Sample T (Unequal Variance)	$A=\sqrt{\frac{S_1^2}{n_1}}\ldots B=\sqrt{\frac{S_2^2}{n_2}}\ldots\ldots\ldots S_{\bar{X}_1-\bar{X}_2}=\sqrt{A+B}$ $df=\frac{(A+B)^2}{\frac{A^2}{(n_1-1)}+\frac{B^2}{(n_2-1)}}\ldots\ldots t=\frac{\bar{X}_1-\bar{X}_2}{S_{\bar{X}_1-\bar{X}_2}}$	$S^2{}_1$ = Standard Deviation of the Sample One N_1 = Sample, number one $S^2{}_2$ = Standard Deviation of the Sample Two N_2 = Sample, number two X_1 = Average of Sample One X_2 = Average of Sample Two
2t Test of Means (separate s)	$t_{n-1,1-\alpha/2}=\frac{\bar{X}_1-\bar{X}_2}{\sqrt{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)}}$	
2t Test of Means (pooled s)	$t_{n-1,1-\alpha/2}=\frac{\bar{X}_1-\bar{X}_2}{S_p\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$	
2Z Test of Means (Equal Variance)	$z_{1-\alpha/2}=\frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{\sigma^2{}_1}{n_1}+\frac{\sigma^2{}_2}{n_2}}}$	
AOV Sum of Squares Correction Factor	$C=\frac{T^2}{N}$	
AOV Sum of Squares Total	$SS_{Total}=\sum X^2-\frac{(\sum X)^2}{n}$	
AOV Sum of Squares Total (Calculator)	$SS_{Total}=(n-1)s^2_{Total}$	

Calculation	Formula	Notes
Chi Squared (Variance, not Proportions)	$\chi^2 = \frac{(n-1)s^2}{\sigma_o^2}$	
F Statistic (Variances)	$\frac{1}{F_{\alpha/2;\nu Y;\nu X}(s_x^2/s_y^2)} \leq \frac{\sigma_Y^2}{\sigma_x^2} \leq \frac{F_{\alpha/2;\nu X;\nu Y}}{(s_x^2/s_y^2)}$	
Interval for Proportions	$p_1 - p_2 \pm z_{1-\alpha/2}\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	
Paired t Test	$t_{n-1,1-\alpha/2} = \frac{\bar{d}}{s_d/\sqrt{n}}$	
Pooled Variance	$s_p = \sqrt{\frac{(n_1-1)s^2_1 + (n_2-1)s^2_2}{n_1+n_2-2}}$	
Single Sample Test of Proportions	$z_{1-\alpha/2} = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$	
Single Sample (Compare to Standard)	$t_{n-1,1-\alpha/2} = \frac{\overline{X}-\mu}{\frac{S}{\sqrt{n}}}$	
Sum of Squares (Equal n)	$SS_{Treatment} = n(k-1)s_{\bar{x}}^2$	

Calculation	Formula	Notes
Sum of Squares (Unequal n)	$SS_{Treatment} = \sum_{i=1}^{k} n_i \bar{x}_i^2 - N\bar{\bar{x}}^2$	
Sum of Squares Treatments	$SS_{Treatments} = \sum_{i-1}^{k} \frac{T_i^2}{n_i} - C$	
Two Sample Test of Proportions	$z_{1-\alpha/2} = \frac{p_1 - p_2}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$	
Variance	$F_{n_1-1,n_2-1,1-\alpha} = \frac{s^2{}_1}{s^2{}_2}$	

Regression

Calculation	Formula	Notes
Correlation Coefficient	$r = \frac{\sum (X_m - \overline{X})(Y_m - \overline{Y})}{\sqrt{\sum (X_m - \overline{X})^2 \sum (Y_m - \overline{Y})^2}}$	
Linear Regression	$\hat{Y} = a + bX$	
Multiple Regression	$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_3 ... + b_kX_k$	
Multiple Standard Error	$SE = \sqrt{\frac{\sum \left(Y - \hat{Y}\right)^2}{n - (k+1)}}$	

Capability Indices

Calculation	Formula	Notes
Confidence Interval for C_{pk}	$C_{pk} \pm Z_{\alpha/2}\sqrt{\frac{1}{9n}+\frac{C_{pk}^{\ 2}}{2n-2}}$	
C_{pk} Attribute Data	$C_{pk} = \frac{1.5 + Z_{P(Good)}}{3}$	
Process Capability	$C_p = \frac{(SpecLimit_{Upper}) - (SpecLimit_{Lower})}{6s}$	
Process Centering (Short Term 1)	$C_{pk} = Minimum \left\{ \frac{Z_{Upper}}{3}, \frac{Z_{Lower}}{3} \right\}$	
Process Centering (Short Term 2)	$Z_{Upper} = \frac{(SpecLimit_{Upper} - \overline{X})}{s}$ $Z_{Lower} = \frac{(\overline{X} - SpecLimit_{Lower})}{s}$	
Sigma of C_{pk}	$\sigma_{Cpk} = \sqrt{\frac{1}{9n} + \frac{C_{pk}^{\ 2}}{2n-2}}$	

Control Charts

Calculation	Formula	Notes
c Chart (Fixed n_1)	$c = \#\ Defects$ $\bar{c} = \frac{\sum c_i}{\sum n_i}$	
c Chart (Fixed n_2)	$UCL = \bar{c} + 3\sqrt{\bar{c}}$ $LCL = \bar{c} - 3\sqrt{\bar{c}}$	
Control Limits for n_p	$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ $LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	
np Chart (Fixed n)	$n\bar{p} = \frac{\sum np}{\#\ Subgroups}$	
p Chart (Average Sample Size)	$\bar{n} = \frac{\sum n_i}{k}$	
p Chart (n can vary, Control Limits for Proportions)	$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$ $LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$	
p Chart (n can vary, Mean Percent Defects)	$Subgroup\ p = \frac{np}{p}$ $\bar{p} = \frac{\sum np}{\sum n}$	

Calculation	Formula	Notes
u Chart (n can vary, Mean Percentage Defectives)	$Subgroup = u = \frac{c}{n}$ $\bar{u} = \frac{\sum c}{\sum n}$	
u Chart (Average Sample Size)	$\bar{n} = \frac{\sum n_i}{k}$	
u Chart (n can vary, Control Limits for Proportions)	$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$ $LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$	
X and R Charts (Grand Mean)	$\bar{\bar{X}} = \frac{\sum \bar{X}}{k}$	
X and R Charts (Control Limits for the Mean)	$UCL = \bar{\bar{X}} + A_2\bar{R}$ $LCL = \bar{\bar{X}} - A_2\bar{R}$	
X and R Charts (Control Limits for the Range)	$UCL = D_4\bar{R}$ $LCL = D_3\bar{R}$	
X and R Charts (Range Target)	$\bar{R} = \frac{(R_1 + R_2 \ldots R_n)}{k}$	

Calculation	Formula	Notes
X and S Charts (Control Limits for the Mean)	$UCL = \overline{\overline{X}} + A_3\overline{S}$ $LCL = \overline{\overline{X}} - A_3\overline{S}$	
X and S Charts (Grand Mean)	$\overline{\overline{X}} = \frac{\sum \overline{X}}{k}$	
X and S Charts (Grand Mean)	$\overline{S} = \frac{(S_1 + S_2 \ldots S_n)}{k}$	
X and S Charts (Control Limits for the Range)	$UCL = B_4\overline{S}$ $LCL = B_4\overline{S}$	

Z Table

Normal Distribution - Right Tail Area

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
4.0	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002
4.1	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00001
4.2	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
4.3	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
4.4	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
4.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
4.6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
4.7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
4.8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
4.9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

T Table

Alpha Risk → / DF = n-1 ↓	0.600	0.700	0.800	0.900	0.950	0.975	0.990	0.995
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576

F Distribution (Table 1)

Tabulated values for a = .05

D/N	1	2	3	4	5	6	7	8	9	10
1	161.40	199.50	215.70	224.60	230.20	234.00	236.80	238.90	240.50	241.90
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

F Distribution (Table 2)

Tabulated values for a = .05

D/N	12	15	20	24	30	40	60	120	
1	243.90	245.90	248.00	249.10	250.10	251.10	252.20	253.30	254.30
2	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Chi- Squared Distribution (Table 1)

df	0.250	0.100	0.050	0.025	0.010	0.005	0.001
1	1.323	2.706	3.841	5.024	6.635	7.879	10.828
2	2.773	4.605	5.991	7.378	9.210	10.597	13.816
3	4.108	6.251	7.815	9.348	11.345	12.838	16.266
4	5.385	7.779	9.488	11.143	13.277	14.860	18.467
5	6.626	9.236	11.070	12.832	15.086	16.750	20.515
6	7.841	10.645	12.592	14.449	16.812	18.548	22.458
7	9.037	12.017	14.067	16.013	18.475	20.278	24.322
8	10.219	13.362	15.507	17.535	20.090	21.955	26.125
9	11.389	14.684	16.919	19.023	21.666	23.589	27.877
10	12.549	15.987	18.307	20.483	23.209	25.188	29.588
11	13.701	17.275	19.675	21.920	24.725	26.757	31.264
12	14.845	18.549	21.026	23.337	26.217	28.300	32.909
13	15.984	19.812	22.362	24.736	27.688	29.819	34.528
14	17.117	21.064	23.685	26.119	29.141	31.319	36.123
15	18.245	22.307	24.996	27.488	30.578	32.801	37.697
16	19.369	23.542	26.296	28.845	32.000	34.267	39.252
17	20.489	24.769	27.587	30.191	33.409	35.718	40.790
18	21.605	25.989	28.869	31.526	34.805	37.156	43.312
19	22.718	27.204	30.144	32.852	36.191	38.582	43.820
20	23.828	28.412	31.410	34.170	37.566	39.997	45.315
21	24.935	29.615	32.671	35.479	38.932	41.401	46.797
22	26.039	30.813	33.924	36.781	40.289	42.796	48.268
23	27.141	32.007	35.172	38.076	41.638	44.181	49.728
24	28.241	33.196	36.415	39.364	42.980	45.558	51.179
25	29.339	34.382	37.652	40.646	44.314	46.928	52.620
26	30.434	35.563	38.885	41.923	45.642	48.290	54.052
27	31.528	36.741	40.113	43.194	46.963	49.645	55.476
28	32.620	37.916	41.337	44.461	48.278	50.993	56.892
29	33.711	39.087	42.557	45.722	49.588	52.336	58.302
30	34.800	40.256	43.773	46.979	50.892	53.672	59.703
40	45.616	51.805	55.758	59.342	63.691	66.766	73.402
50	56.334	63.167	67.505	71.420	76.154	79.490	86.661
60	66.981	74.397	79.082	83.298	88.379	91.952	99.607
70	77.577	85.527	90.531	95.023	100.425	104.215	112.317
80	88.130	96.578	101.879	106.629	112.329	116.321	124.839
90	98.650	107.565	113.145	118.136	124.116	128.299	137.208
100	109.141	118.498	123.342	129.561	135.807	140.169	149.449

Chi- Squared Distribution (Table 2)

df	0.995	0.990	0.975	0.950	0.900	0.750	0.500
1	0.000	0.000	0.001	0.004	0.158	0.102	0.455
2	0.010	0.020	0.051	0.103	0.211	0.575	1.386
3	0.072	0.115	0.216	0.352	0.584	1.213	2.366
4	0.207	0.297	0.484	0.711	1.064	1.923	3.357
5	0.412	0.554	0.831	1.145	1.610	2.675	4.351
6	0.676	0.872	1.237	1.635	2.204	3.455	5.348
7	0.989	1.239	1.690	2.167	2.833	4.255	6.346
8	1.344	1.646	2.180	2.733	3.490	5.071	7.344
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342
11	2.603	3.053	3.816	4.575	5.578	7.584	10.341
12	3.074	3.571	4.404	5.226	6.304	8.438	11.340
13	3.565	4.107	5.009	5.892	7.042	9.299	12.340
14	4.075	4.660	5.629	6.571	7.790	10.165	13.339
15	4.601	5.229	6.262	7.261	8.547	11.036	14.339
16	5.142	5.812	6.908	7.962	9.312	11.912	15.338
17	5.697	6.408	7.564	8.672	10.085	12.792	16.338
18	6.265	7.015	8.231	9.390	10.865	13.675	17.338
19	6.844	7.633	8.907	10.117	11.651	14.562	18.338
20	7.434	8.260	9.591	10.851	12.443	15.452	19.337
21	8.034	8.897	10.283	11.591	13.240	16.344	20.337
22	8.643	9.542	10.982	12.338	14.041	17.240	21.337
23	9.260	10.196	11.688	13.091	14.848	18.137	22.337
24	9.886	10.856	12.401	13.848	15.659	19.037	23.337
25	10.520	11.524	13.120	14.611	16.473	19.939	24.337
26	11.160	12.198	13.844	15.379	17.292	20.843	25.336
27	11.808	12.879	14.573	16.151	18.114	21.749	26.336
28	12.461	13.565	15.308	16.928	18.939	22.657	27.336
29	13.121	14.256	16.047	17.708	19.768	23.567	28.336
30	13.787	14.953	16.791	18.493	20.599	24.478	29.336
40	20.707	22.164	24.433	26.509	29.051	33.660	39.335
50	27.991	29.707	32.357	34.764	37.689	42.942	49.335
60	35.535	37.485	40.482	43.188	46.459	52.294	59.335
70	43.275	45.442	48.758	51.739	55.329	61.698	69.334
80	51.172	53.540	57.153	60.391	64.278	71.145	79.334
90	59.196	61.754	65.647	69.126	73.291	80.625	89.334
100	67.328	70.065	74.222	77.929	82.358	90.133	99.334